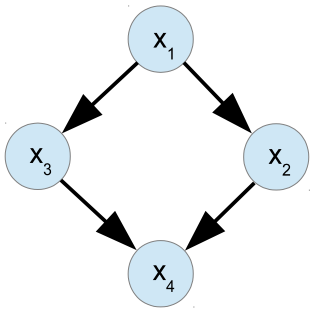
ML2, Summer 2020,
Exercise 2, Supervised structured learning

2.1. The joint probability in the variables x1,..., x7 shall be given as
p(x1, x2, x3, x4, x5, x6, x7) = p(x1)p(x3)p(x7)p(x2|x1,x3)p(x5|x7, x2)p(x4|x2)p(x6|x5, x4).
Draw the directed graphical model for this joint probability!

2.2. Write down the joint probability for the DGM given in the pictures:



2.3. Max marginal inference in chains

A Bus goes from your home to the university where you leave it at the 4th stop.
With certain probabilities the bus arrives on time a the stops x_i. Calculate these probabilities given the directed graphical model (DGM):

Algorithm for marginal computation ("Sum-Product Message Passing")
1. Compute Messages from right to left ⇦
2.. Read out all marginals

and the following (conditional) probability tables $p(x_i|x_{i-1})$ (x=0 means late, x=1 on time):

x1	
0	0,29
1	0,71

x2|x1 x2	x1 = 0	x1 = 1
0	0,39	0,2
1	0,61	0,8

x3|x2 x3	x2 = 0	x2 = 1
0	0,77	0,51
1	0,23	0,49

x4|x3 x4	x3 = 0	x3 = 1
0	0,11	0,66
1	0,89	0,34

(equivalently: What is the maximum marginals solution of this probability distribution?)

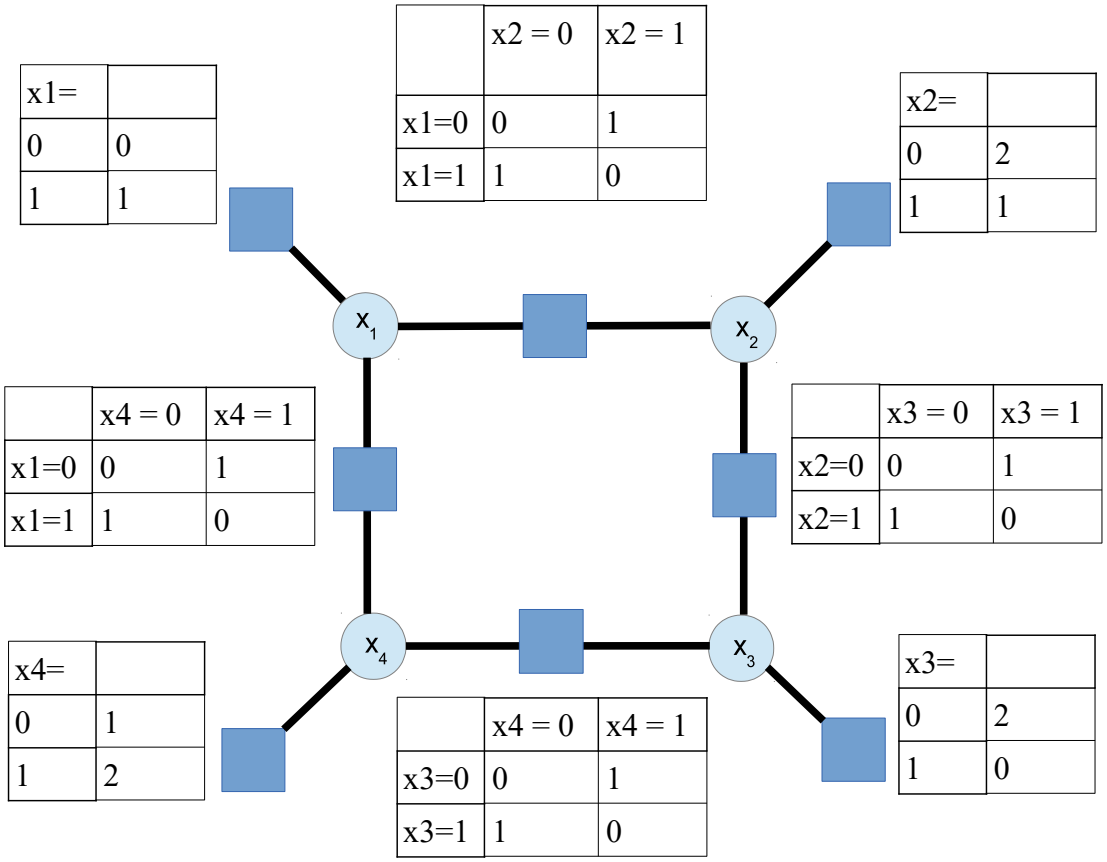
2.4. Factor graphs
For the chain in 2.2.: Draw the factor graph and write down the formula that corresponds to this factor graph. The conditional probabilities from the directed graphical model are inserted as factors here.

2.5. Let a factor be

$\psi(x_1, x_2) = \begin{Bmatrix} x_1 + x_2 \mid 0 \leqslant x_1, x_2 \leqslant 2 \\ 0 \quad \mid x_1, x_2 < 0 \,;\, x_1, x_2 > 2 \end{Bmatrix}$. There are no more factors.

Which distribution $p(x_1, x_2)$ follows from that factor?

2.6. Given is the following energy function
(of a Gibbs-Boltzmann-Distribution):



Calculate the MAP solution with ICM.